Factoring Summary

1. Factor out the Greatest Common Factor (GCF)

Form for GCF: \( ax^2 + ax + ab = a(x^2 + x + b) \)

2. Factor by grouping (see example below)

3. Form: \( x^2 + bx + c \). Find factors of \( c \) that add to get \( b \) and multiply to get \( c \).

4. Form: \( ax^2 + bx + c \). Use trial and error to find the factored form.

5. Form: \( P^2 \pm 2PQ + Q^2 \). Then this factors into: \( (P \pm Q)^2 \) (called “perfect square”)

6. Form: \( x^2 - y^2 = (x+y)(x-y) \)

7. Form: \( x^2 + y^2 \) (cannot be factored with integers!)

8. Form: \( x^3 + y^3 = (x+y)(x^2 - xy + y^2) \)

9. Form: \( x^3 - y^3 = (x-y)(x^2 + xy + y^2) \)

Examples: Directions - factor all of the following completely.

1. \( 3x^2 + 9x + 15 \) has a GCF of 3. (NOTE: ALWAYS FACTOR OUT GCF FIRST!!)
   Thus, factoring out 3 yields: \( 3(x^2 + 3x + 5) \)
   (Since the expression inside the parantheses cannot be factored, this is the final answer.)

2. \( 3x^3 + 2x^2 - 6x - 4 \) is a candidate for factoring by grouping. Grouping terms:
   \( (3x^3 + 2x^2) + (-6x - 4) = x^2(3x + 2) - 2(3x + 2) = (3x + 2)(x^2 - 2) \)

3. \( x^2 + 4x - 12 \) Since \( a=1 \) in the trinomial, need to find factors of -12 that add to get 4.
   All the possible pairs of factors for -12 are: \( 1, -12; -1, 12; 2, -6; -2, 6; 3, -4; -3, 4 \)
   Since the only pair that adds to 4 is \(-2,6\) the answer is: \( (x - 2)(x + 6) \)

4. \( 3x^2 + 2x - 8 \) Since \( a\neq1 \) in the trinomial, use trial and error to find the answer.
   The factor pairs of 3 are: \([3,1]\).
   The factor pairs for \(-8\) are: \([1,-8], [-1,8], [2,-4], [-2,4]\)
   By trial and error it is found that the answer is: \( (3x - 4)(x + 2) \)

<table>
<thead>
<tr>
<th>If trinomial has the form:</th>
<th>Sign Hints:</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ax^2 + bx + c )</td>
<td>then factored form is ((px + m)(qx + n))</td>
</tr>
<tr>
<td>( ax^2 - bx + c )</td>
<td>then factored form is ((px - m)(qx - n))</td>
</tr>
<tr>
<td>( ax^2 \pm bx - c )</td>
<td>then factored form is ((px + m)(qx - n) \text{ OR } (px - m)(qx + n))</td>
</tr>
</tbody>
</table>

5. \( 4x^2 - 12x + 9 \) is in the form \( P^2 \pm 2PQ + Q^2 \). Thus, \( 4x^2 - 12x + 9 = (2x - 3)^2 \) (perfect square)

6. \( 9x^2 - 36y^2 \) is in the form of \( x^2 - y^2 = (x+y)(x-y) \). Thus, \( 9x^2 - 36y^2 = (3x + 6y)(3x - 6y) \)

7. \( 9x^2 + 36y^2 \) is in the form of \( x^2 + y^2 \). Thus, \( 9x^2 + 36y^2 \) is non-factorable using integers.

8. \( 8x^3 + 27y^3 \) is in the form of \( x^3 + y^3 = (x+y)(x^2 - xy + y^2) \)
   Thus, \( 8x^3 + 27y^3 = (2x + 3y)(4x^2 - 6xy + 9y^2) \)

9. \( 8x^3 - 27y^3 \) is in the form of \( x^3 - y^3 = (x-y)(x^2 + xy + y^2) \)
   Thus, \( 8x^3 - 27y^3 = (2x - 3y)(4x^2 + 6xy + 9y^2) \)

This instructional aid was prepared by the Tallahassee Community College Learning Commons.